Solar Collector Analysis and Design

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Alternative Energy Engineering II

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Outline

- Midterm exam solutions
- Types of solar collectors
- Review Heat Transfer Basics
- Overview of solar collector analysis
- Analysis of losses from solar collectors
- Transfer of net heat gain to fluid tubes
- Increase in temperature of fluid
- Design equation for solar collectors

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Midterm Exam

- Gas turbine using landfill gas with 85% CH_4 (Q_c = 802,802 kJ/kMol) 15% CO_2, 5% N_2
- $P_{comp,in}$ = 100 kPa; $T_{comp,in}$ = 290 K; $\eta_{s,comp}$ = 87%; $DP_{combustor}$ = 30 kPa; $T_{in,turbine}$ = 1450 K; $P_{out,turb}$ = 110 kPa; $\eta_{s,turb}$ = 90%; P = 50 MW, $\eta_{generator}$ = 94%
- Find: air and fuel mass flows and outlet $\%O_2$
- Start by finding heating value of landfill gas

$$Q_c = \frac{\overline{Q}_c}{M} = \frac{\sum \omega_i \overline{Q}_i}{\sum \omega_i M_i} = \frac{(0.8)\left(\frac{802{,}802\,kJ}{kMol}\right) + (0.2)0}{(0.8)\left(\frac{16.04\,kg}{kMol}\right) + (0.15)\left(\frac{44.01\,kg}{kMol}\right) + (0.05)\left(\frac{28.01\,kg}{kMol}\right)} = \frac{30{,}827\,kJ}{kg}$$

Denominator is M_{fuel} = 20.83 kg/kmol

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Midterm Problem One II

- Compute compressor and turbine

$$T_2 = T_1\left\{1 + \frac{1}{\eta_c}\left[\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1\right]\right\} = 290\,K\left\{1 + \frac{1}{0.87}\left[10^{\frac{1.4-1}{1.4}} - 1\right]\right\} = 600.2\,K$$

$$T_4 = T_3\left\{1 + \eta_t\left[\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} - 1\right]\right\} = 1473.15\,K\left\{1 + 0.88\left[\left(\frac{110\,kPa}{970\,kPa}\right)^{\frac{1.334-1}{1.334}} - 1\right]\right\} = 901.7\,K$$

$$w_c = \overline{c}_{p,compressor}(T_1 - T_2) = \frac{1.004\,kJ}{kg \cdot K}(290\,K - 600.2\,K) = \frac{-311.5\,kJ}{kg}$$

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Midterm Problem One III

$$w_t = \overline{c}_{p,turbine}(T_3 - T_4) = \frac{1.148\,kJ}{kg \cdot K}(1450\,K - 901.7\,K) = \frac{629.5\,kJ}{kg}$$

- Get fuel/air ratio from combustor

$$\frac{\dot{m}_{fuel}}{\dot{m}_{air}} = \frac{\overline{c}_{p,combustor}(T_3 - T_2)}{Q_c} = \frac{\frac{1.076\,kJ}{kg \cdot K}(1450\,K - 600.2\,K)}{\frac{30{,}827\,kJ}{kg}} = 0.02966$$

$$\dot{W}_{net} = (\dot{m}_{air} + \dot{m}_{fuel})w_t + \dot{m}_{air}w_c = \dot{m}_{air}\left[\left(1 + \frac{\dot{m}_{fuel}}{\dot{m}_{air}}\right)w_t + w_c\right]$$

$$53{,}191\,kW\frac{1\,kJ}{kW \cdot s} = \dot{m}_{air}\left[(1 + 0.02966)\frac{629.5\,kJ}{kg} + \frac{-311.5\,kJ}{kg}\right]$$

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Midterm Problem One IV

$$\dot{m}_{air} = \frac{158.0\,kg}{s} \qquad \dot{m}_{fuel} = \frac{fuel}{air}\dot{m}_{air} = (0.02966)\frac{158.0\,kg}{s} = \frac{4.686\,kg}{s}$$

- **Find:** $\%O_2$ in exhaust
  – Need to find fuel formula components, A and λ
  – Get fuel formula as average over all componentds

$$\%\,O_2 = \frac{100(\lambda - 1)A}{D} = \frac{100\,(\lambda - 1)A}{x + \lambda\,A\,B_d - A + z + \frac{v}{2}}$$

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Midterm Problem One V

- x = (0.8)(1) + (0.15)(1) + (0.05)(0) = 0.95
- y = (0.8)(4) + (0.15)(0) + (0.05)(0) = 3.20
- w = (0.8)(0) + (0.15)(2) + (0.05)(0) = 0.30
- v = (0.8)(0) + (0.15)(0) + (0.05)(2) = 0.10
- A = x + y/4 – w/2 = 0.95 + 3.20/4 – 0.30/2 = 1.6
- Get λ from Air/Fuel ratio found previously
  - Air/Fuel = 1/(fuel/air) = 1/0.02966 = 31.71

$$\frac{Air}{Fuel} = = \frac{138.27\lambda A}{M_{Fuel}} \Rightarrow \lambda = \frac{Air}{Fuel}\frac{M_{Fuel}}{138.27A} = \frac{31.71\,kg\;air}{kg\;fuel}\frac{\frac{20.83\,kg\;fuel}{kmol\;fuel}}{\frac{138.27\,kg\;air}{kmol\,O_2}\frac{1.6\,kmol\,O_2}{kmol\;fuel}} = 3.175$$

$$\%O_2 = \frac{100(\lambda-1)A}{x+\lambda A B_d - A + z + \frac{v}{2}} = \frac{100(3.175-1)1.6}{.95+2.175(1.6)(4.7742)-1.6+0+\frac{0.1}{2}}$$

%O2 = 14.71%

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Midterm Problem Two

- **Given:** Landfill-gas gas turbine produces 50 MW electricity at 80% capacity factor; maintenance cost = \$0.007/kWh; sales price = \$0.06/kWh
- **Find:** Cost to return 9% for 23 years
- 50 MW and 80% capacity factor gives (50,000 kW)(80%)(8766 h/yr) = 3.506x10^8 kWh/yr
- Sales minus maintenance = \$0.06/kWh – \$0.07/kWh = \$0.053/kWh
- Annual income = (3.506x108 kWh/yr)(\$0.053/kWh) = \$1.858x107/yr

$$P = \frac{\$1{,}858x10^7}{yr}\left(\frac{P}{A}\right) = \frac{\$1{,}858x10^7}{yr}\left(\frac{1-(1+i)^{-N}}{i}\right) = \frac{\$1{,}858x10^7}{yr}\left(\frac{1-(1+.09)^{-N}}{.09/yr}\right) = \$1.78x10^8$$

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Midterm Problem Three

- **Given:** Discrete wind speed data

Percent of Wind-Speed Data Between Lower and Upper Velocity Bounds (V in m/s)								
Lower	Upper	Percent	Lower	Upper	Percent	Lower	Upper	Percent
0	1	2.8747%	10	11	4.3213%	20	21	0.8028%
1	2	9.8109%	11	12	4.1559%	21	22	0.5310%
2	3	10.307%	12	13	4.1527%	22	23	0.3928%
3	4	9.4960%	13	14	3.9050%	23	24	0.2427%
4	5	8.0058%	14	15	4.0583%	24	25	0.1476%
5	6	6.0967%	15	16	3.4830%	25	26	0.1102%
6	7	5.1868%	16	17	3.0287%	26	27	0.0716%
7	8	4.6691%	17	18	2.1695%	27	28	0.0310%
8	9	4.6374%	18	19	1.6005%	28	29	0.0114%
9	10	4.3865%	19	20	1.2489%	29		0.0640%

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Midterm Problem Three II

- **(a) show: Relation for mean cubed velocity in one band between V = a and V = b**

$$\overline{V^3}_{band} = \int_a^b \frac{1}{b-a}V^3 dV = \frac{1}{b-a}\left.\frac{V^4}{4}\right|_a^b = \frac{(b^4-a^4)}{4(b-a)} =$$

$$\frac{(b^2-a^2)(b^2+a^2)}{4(b-a)} = \frac{(b+a)(b-a)(b^2+a^2)}{4(b-a)} = \frac{(b+a)(b^2+a^2)}{4}$$

- **(b) Find: contribution of wind speeds between rated and cut-out speeds to the average operating power**
- This is P_{max} times fraction of speeds in this range

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Midterm Problem Three III

$$V_{P\max} = \left(\frac{2P_{\max}}{c_p\rho A}\right)^{\frac{1}{3}} = \left[\frac{2(2.5x10^6 W)\frac{kg\cdot m^2}{W\cdot s^3}}{(0.48)\frac{1.225kg}{m^3}\frac{\pi}{4}(90m)^2}\right]^{\frac{1}{3}} = \frac{11.01m}{s}$$

- sum percentages for all bands from band at a lower limit of 11 m/s to band with an upper limit of 25 m/s: 4.1559% + 4.1527% + 3.9050% + 4.0583% + 3.4830% + 3.0287% + 2.1695% + 1.6005% + 1.2489% + 0.8028% + 0.5310% + 0.3928% + 0.2427% + 0.1476% = 29.9194%.
- P_{max}(0.299) = (2.5 MW)(0.299) = 0.7480 MW

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Midterm Problem Three IV

- **(c) Find: contribution of wind speeds between cut-in speed and rated wind speeds**

$$\overline{P}(V_{cut-in} < V < V_{rated}) = \sum_{k=Start}^{End} f_k \frac{c_p\rho A V_k^3}{2}$$

f_k = fraction in band k

- Start term has cut-in speed as lower bound

$$\frac{(0.060967)(.48)}{2}\frac{1.225kg}{m^3}\frac{\pi}{4}(90m)^2\frac{1}{4}\left(\frac{5m}{s}+\frac{6m}{s}\right)\left[\left(\frac{5m}{s}\right)^2+\left(\frac{6m}{s}\right)^2\right]\frac{1W\cdot s^3}{kg\cdot m^2} = 19{,}128W$$

- End term has rated speed as upper bound

$$\frac{(0.043213)(.48)}{2}\frac{1.225kg}{m^3}\frac{\pi}{4}(90m)^2\frac{1}{4}\left(\frac{10m}{s}+\frac{11m}{s}\right)\left[\left(\frac{10m}{s}\right)^2+\left(\frac{11m}{s}\right)^2\right]\frac{1W\cdot s^3}{kg\cdot m^2} = 93{,}775W$$

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Midterm Problem Three V

- **(d) Find: capacity factor if sum in part (c) = 0.3007 MW**
- Capacity factor = average operating power divided by maximum power
- Average operating power = sum of part (b) and part (c) components = 0.7480 MW + 0.3007 MW = 1.0487 MW
- Capacity factor = (1.0487 MW) / (2.5 MW) = ***41.9%***

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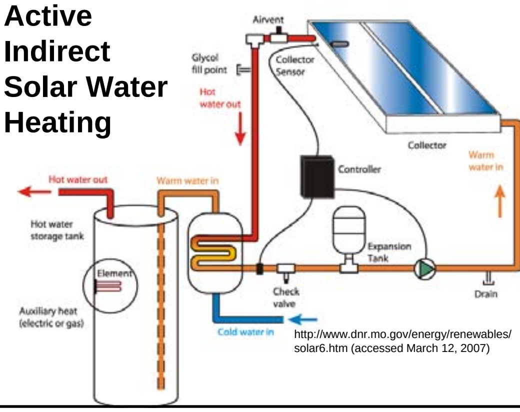
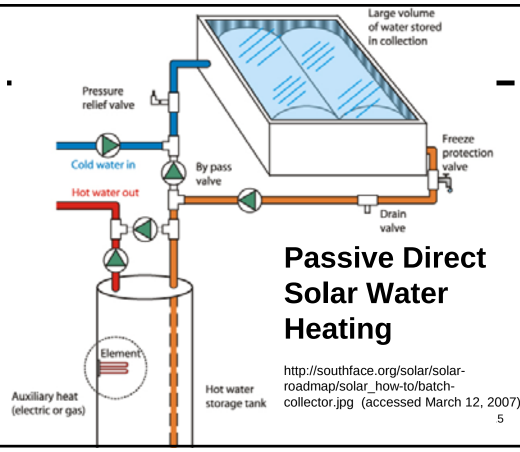
Active Indirect Solar Water Heating



http://www.dnr.mo.gov/energy/renewables/solar6.htm (accessed March 12, 2007)



Passive Direct Solar Water Heating

http://southface.org/solar/solar-roadmap/solar_how-to/batch-collector.jpg (accessed March 12, 2007)

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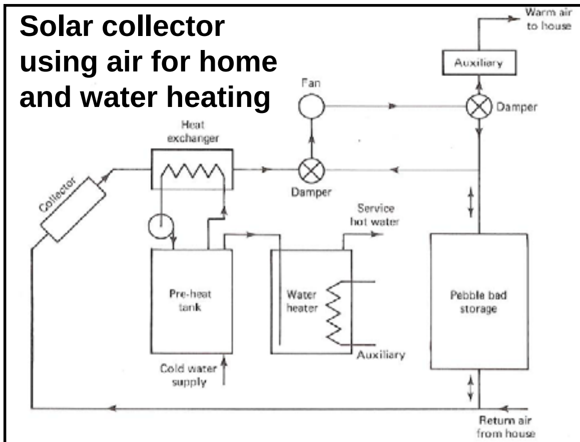
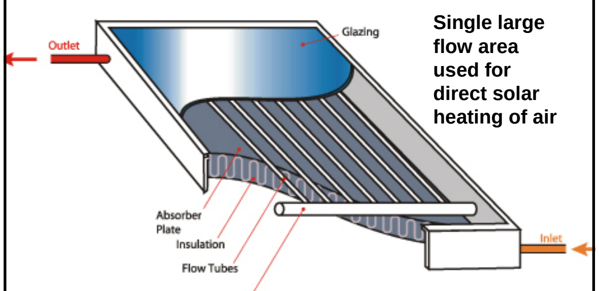
Solar collector using air for home and water heating



Liquid Flat Plate Collector



Single large flow area used for direct solar heating of air

http://southface.org/solar/solar-roadmap/solar_how-to/solar-how_solar_works.htm accessed March 12, 200

Review Fourier's Law

- Basic law for heat conduction
- Actually a vector equation $\dot{q} = -k \text{ grad } T$
- k is thermal conductivity
  - Units of k are W/m·K or Btu/hr·ft·R
- For one dimensional heat transfer, $\dot{q}_x = -k dT/dx$; integration (constant $\dot{q}_x$) gives

$$\dot{q} = \frac{k(T_1 - T_2)}{L} \quad or \quad \dot{Q} = \dot{q}A = \frac{kA(T_1 - T_2)}{L}$$

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Review Convection Basics

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

h = heat transfer coefficient (W/m²·K) or Btu/hr·ft²·ºF

h is found from empirical or theoretical equation

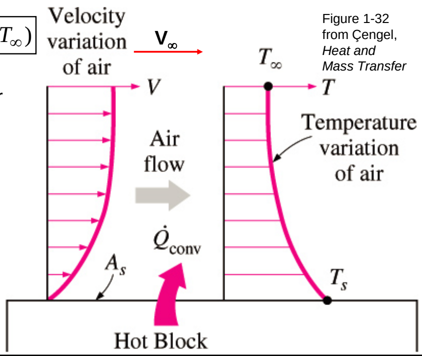

Figure 1-32 from Çengel, *Heat and Mass Transfer*

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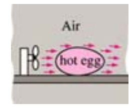
Review Convection Types

- **Free (natural) convection** comes from buoyancy, **forced convection** has a driven flow
- Flows contained in pipes and ducts are **internal flows**; egg pictures show **external flow**
- Other considerations are **laminar *vs.* turbulent** flow and convection during **boiling or condensation**



Eggs from Figure 1-33 in Çengel, *Heat and Mass Transfer*

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Review Thermal Resistance

- Heat flow analogous to current
- Temperature difference analogous to potential difference
- Both follow Ohm's law with appropriate resistance term
- Current: $I = (V_1 - V_2) / R$
- Heat Transfer: $Q = (T_1 - T_2) / R_{thermal}$

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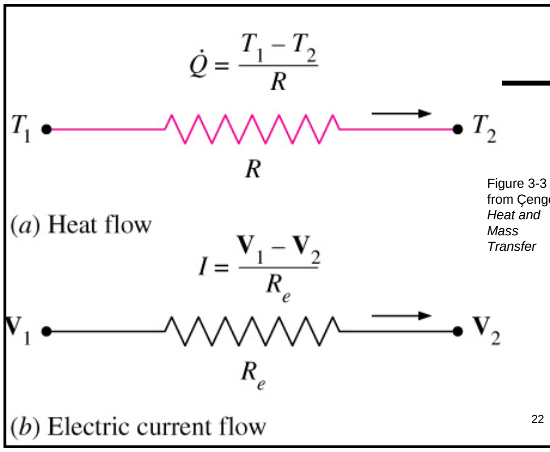

Figure 3-3 from Çengel, *Heat and Mass Transfer*

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Review Thermal Resistance II

- Conduction

$$\dot{Q} = \frac{\bar{k}A(T_1 - T_2)}{L} \Rightarrow \dot{Q} = \frac{T_1 - T_2}{R_{cond}} \Rightarrow R_{cond} = \frac{L}{kA}$$

- Convection

$$\dot{Q} = hA(T_s - T_f) \Rightarrow \dot{Q} = \frac{T_s - T_f}{R_{conv}} \Rightarrow R_{conv} = \frac{1}{hA}$$

- Radiation

$$R_{rad} = \frac{1}{A_1\mathfrak{F}_{12}\sigma(T_1^3 + T_2^3 + T_2^2T_1 + T_1^2T_2)} = \frac{1}{A_1h_{rad}}$$

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Review Thermal Circuits

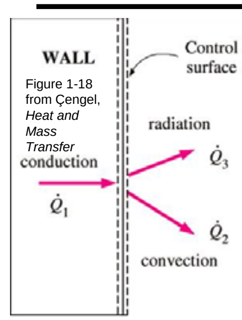

Figure 1-18 from Çengel, *Heat and Mass Transfer*

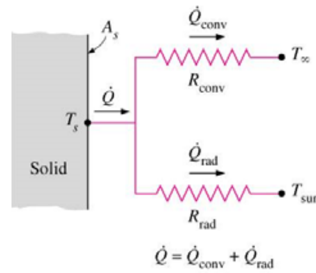

Figure 3-5 from Çengel, *Heat and Mass Transfer*

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Parallel Resistances ($T_\infty = T_{surr}$)

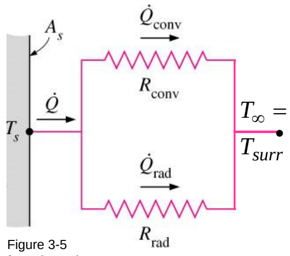

Figure 3-5 from Çengel, *Heat and Mass Transfer*

$$\frac{1}{R_{total}} = \frac{1}{R_{conv}} + \frac{1}{R_{rad}}$$

$$\frac{1}{R_{total}} = A_s h_{conv} + A_s h_{rad}$$

Define total heat transfer coefficient, h_{total}

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} \qquad h_{total} = \frac{1}{A_s R_{total}} = h_{conv} + h_{rad}$$

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Review Combined Modes

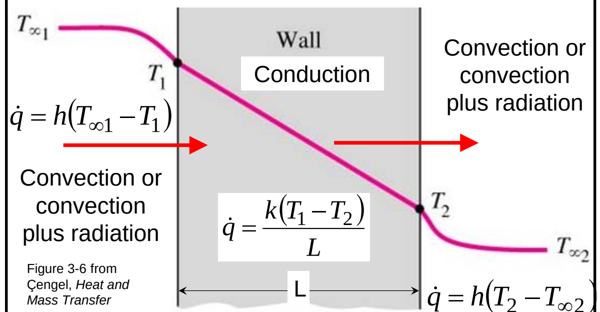

Figure 3-6 from Çengel, *Heat and Mass Transfer*

All $\dot{q}$ values are the same

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Review Combined Modes II

A is area normal to heat flow

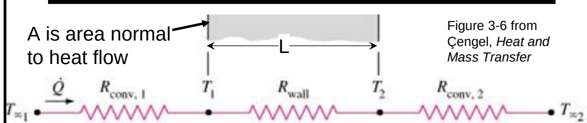

Figure 3-6 from Çengel, *Heat and Mass Transfer*

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{conv,1} + R_{wall} + R_{conv,1}}$$

Series Resistance Formula

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{Ah_1} + \frac{L}{kA} + \frac{1}{Ah_2}} \Rightarrow \dot{q} = \frac{\dot{Q}}{A} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}}$$

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Review Combined Modes III

A is area normal to heat flow

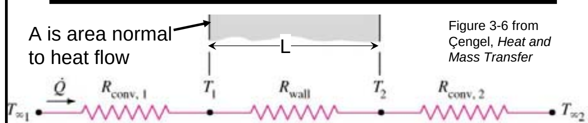

Figure 3-6 from Çengel, *Heat and Mass Transfer*

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}} = \frac{T_{\infty 1} - T_1}{\frac{1}{h_1}} = \frac{T_1 - T_2}{\frac{L}{k}} = \frac{T_2 - T_{\infty 2}}{\frac{1}{h_2}}$$

Can analyze each step individually, but heat transfer is the same for all series steps

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Basic Solar Collector Analysis

- Overall heat balance
  - Incoming solar radiation
  - Heat loss from collector to environment
  - Useful energy gain = Incoming Solar Radiation – Environmental Heat Loss
- Environmental heat loss proportional to $\Delta T = T_{collector} - T_{ambient}$
  - Applications that require high collector temperatures will have more heat loss

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Useful Heat Transfer

- Heat is added to a collector fluid
  - Typically collector fluid is water or water and anti-freeze solution
  - Air is also used as collector fluid for home heating
- Energy added from simple first law for open system with consant pressure heat addition

$$\dot{Q}_u = \dot{m} c_p \left(T_{f,out} - T_{f,in}\right)$$

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Solar to Useful Energy

- Solar transmission through glass covers provides absorbed radiation, H_a
- Consider three losses
  - – Conduction through bottom of solar collector box
  - – Conduction through edge of box
  - – Loss through top
    - Convection between absorber plate and glass covers with conduction through glass
    - Convection from top glass cover to ambient

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Solar to Useful Energy II

- Absorbed radiation minus losses different for different areas of collector
- Fluid temperature increases from inlet to outlet
- What is fluid temperature increase from absorbed radiation minus losses?
- Reference: *Solar Energy Engineering* by Jui Hsieh, Prentice-Hall 1986

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Loss Through Top

- In steady state the following heat rates will be the same
  - – Between absorber plate and bottom glass
  - – From bottom glass to top glass
    - Consider two-plate collector
  - – From top glass to ambient
  - – Look at exchange between absorber plate at temperature T_P and bottom glass at temperature T_{g2}
  - – Have convection plus radiation

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Loss Through Top II

$$Q_{top} = h_{p-g2} A_c (T_P - T_{g2}) + \frac{A_c \sigma (T_P^4 - T_{g2}^4)}{\frac{1}{\varepsilon_P} + \frac{1}{\varepsilon_{g2}} - 1}$$

- Equation for convection plus radiation between two parallel plates
  - – A_c = Collector Area
  - – h_{p-g2} = convection coefficient for air gap
  - – ε_p, ε_{g2} = emissivities of plate and glass
  - – T_P, T_{g2} = temperatures of plate and glass

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Loss Through Top III

$$\frac{A_c \sigma (T_P^4 - T_{g2}^4)}{\frac{1}{\varepsilon_P} + \frac{1}{\varepsilon_{g2}} - 1} = \frac{A_c \sigma (T_P^2 + T_{g2}^2)(T_P + T_{g2})(T_P - T_{g2})}{\frac{1}{\varepsilon_P} + \frac{1}{\varepsilon_{g2}} - 1} = h_{r,p-g2}(T_P - T_{g2})$$

- Define radiation heat transfer coefficient, $h_{r,P-g2}$, as shown in dashed red square
  - – Get equation for heat transfer and thermal resistance, R_{p-g2}

$$Q_{top} = (h_{p-g2} + h_{r,p-g2}) A_c (T_P - T_{g2}) = \frac{T_P - T_{g2}}{R_{p-g2}}$$

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Loss Through Top IV

- Have similar equation for heat transfer between top and bottom glass plate

$$Q_{top} = (h_{g2-g1} + h_{r,g2-g1}) A_c (T_{g2} - T_{g1}) = \frac{T_{g2} - T_{g1}}{R_{g2-g1}}$$

$$h_{r,g2-g1} = \frac{A_c \sigma (T_{g1}^2 + T_{g2}^2)(T_{g1} + T_{g2})}{\frac{1}{\varepsilon_{g1}} + \frac{1}{\varepsilon_{g2}} - 1}$$

  - – T_{g1},ε_{g1} = Temperature, Emissivity of top glass
  - – h_{g2-g1} = convection coefficient for g_1 to g_2

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Loss Through Top V

- Similar equation for heat transfer between top glass plate and ambient

$$Q_{top} = \left(h_{g1-a} + h_{r,g1-a}\right)A_c\left(T_{g1} - T_a\right) = \frac{T_{g1} - T_a}{R_{g2-g1}}$$

$$h_{r,g1-a} = \frac{A_c\sigma\left(T_{g1}^2 + T_{sky}^2\right)\left(T_{g1} + T_{sky}\right)}{\frac{1}{\varepsilon_{g1}} + \frac{1}{\varepsilon_{g2}} - 1}\frac{T_{g1} - T_{sky}}{T_{g1} - T_a}$$

  - Radiation h different here because ΔT for radiation uses T_{sky}, not T_a
  - h_{g1-a} = convection coefficient for g_1 to ambient

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Loss Through Top/Bottom

- Combine three resistances in series to get $R_{top} = R_{P-g2} + R_{g2-g1} + R_{g1-a}$
  - $Q_{top} = (T_P - T_a)/R_{top} = U_{top}A_c(T_P - T_a)$
- Loss through bottom is conduction through insulation (k_{ins}, Δx_{ins}) in series with convection to ambient with h_{b-a}

$$Q_{bottom} = \frac{T_P - T_a}{R_{ins} + R_{conv}} = \frac{T_P - T_a}{\frac{k_{ins}}{\Delta x_{ins}A_c} + \frac{1}{h_{b-a}A_c}} = U_{bottom}A_c\left(T_P - T_a\right)$$

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Total Loss

- $Q_{sides} = U'_{side}A_{side}(T_P - T_a)$
  - Can estimate $U'_{side} = 0.5\ \text{W/m}^2\cdot\text{K}$
  - Use $U_{side}A_c = U'_{side}A_{side}$ for common area
  - $Q_{sides} = U_{side}A_c(T_P - T_a) = (T_P - T_a)/A_{side}$
- Total is sum of individual losses
- $Q_{loss} = U_cA_c(T_P - T_a) = (T_P - T_a)/R_c$
- Overall conductance and resistance
- $U_c = U_{top} + U_{bottom} + U_{sides}$

$$\frac{1}{R_c} = \frac{1}{R_{top}} + \frac{1}{R_{bottom}} + \frac{1}{R_{side}}$$

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Approximate U_c Equation

$$U_c = \frac{1}{\frac{A'}{T_p}\left(\frac{T_p - T_a}{N + B}\right)^{0.33} + \frac{1}{h_w}} + \frac{\sigma\left(T_p + T_a\right)\left(T_p^2 + T_a^2\right)}{\frac{1}{\varepsilon_p + 0.05N\left(1 - \varepsilon_p\right)} + \left(\frac{2N + B - 1}{\varepsilon_g}\right) - N}$$

N = number of glass covers
A' = 250[1 – 0.0044(s – 90)]
s = tilt angle (degrees)
B = (1 – 0.04h_w + 0.0005h_w^2)(1 + 0.091N)
h_w = heat transfer coefficient from top to ambient
Other symbols have previous definitions

Equation uses SI units: U_c and h in W/m²·K, T in K, σ = 5.670x10^{-8} W/m²·K⁴, ε_g is same for all glass covers

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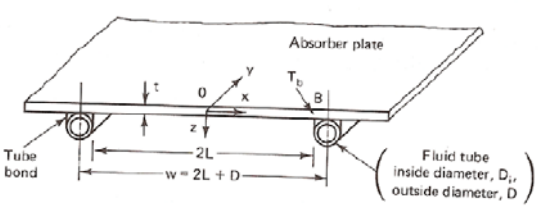
Absorber Plate Analysis



- Small plate thickness, t, gives uniform T in z direction
- Main heat transfer is in x direction with smaller heat transfer in y direction

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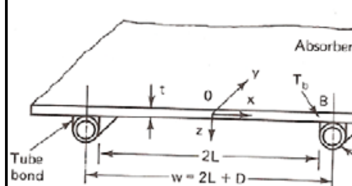
Absorber Plate Analysis II



- Absorber plate is like a fin with dT/dx = 0 at midpoint between tubes (x = 0) and T = T_b at tube bond point (x = ±L)

- Energy balance over δx for unit thickness in y direction has $H_a\delta x$ solar input and loss to ambient = $U_c\delta x(T - T_a)$

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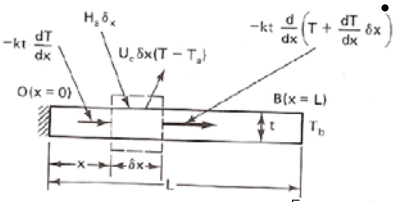
Absorber Plate Analysis III



- Energy balance in x direction conducts net heat (solar input – loss)

$$H_a\delta x - U_c(T-T_a)\delta x - tk\frac{dT}{dx} - \left[-tk\frac{d}{dx}\left(T+\frac{dT}{dx}\delta x\right)\right] = 0$$

$$H_a - U_c(T-T_a) - tk\frac{d^2T}{dx^2} = 0 \qquad \boxed{\frac{d^2T}{dx^2} = \frac{U_c}{tk}\left(T-T_a-\frac{H_a}{U_c}\right)}$$

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Absorber Plate Analysis IV

- Boundary conditions are $T = T_b$ at $x = L$ and $dT/dx = 0$ at $x = 0$
- Define $\theta = T - T_a - H_a/U_c$ so $dT/dx = d\theta/dx$
- $d\theta/dx = 0$ at $x = 0$ and $\theta = T - T_a - H_a/U_c$ at $x = L$ and the differential equation is

$$\frac{d^2T}{dx^2} = \frac{U_c}{tk}\left(T-T_a-\frac{H_a}{U_c}\right) \Rightarrow \frac{d^2\theta}{dx^2} = \theta\frac{U_c}{tk} = m^2\theta$$

$$\theta = A\sinh mx + B\cosh mx \qquad \frac{d\theta}{dx} = Am\cosh mx + Bm\sinh mx$$

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Absorber Plate Analysis V

- For $d\theta/dx = 0$ at $x = 0$

$$0 = \frac{d\theta}{dx} = Am\cosh m0 + Bm\sinh m0 = Am \quad \Rightarrow \quad A = 0$$

- For $\theta = T_b - T_a - H_a/U_c$ at $x = L$

$$T_b - T_P - \frac{H_a}{U_c} = B\cosh mL \qquad B = \frac{T_b - T_a - \frac{H_a}{U_c}}{\cosh mL}$$

$$\theta = B\cosh mx = \frac{T_b - T_a - \frac{H_a}{U_c}}{\cosh mL}\cosh mx$$

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Absorber Plate Analysis VI

$$\frac{\theta}{T_b - T_P - \frac{H_a}{U_c}} = \frac{T - T_a - \frac{H_a}{U_c}}{T_b - T_a - \frac{H_a}{U_c}} = \frac{\cosh mx}{\cosh mL}$$

- Heat flow from plate to tube = $-tkdT/dx|_{x=L}$

$$\left.\frac{dT}{dx}\right|_{x=L} = \left(T_b - T_a - \frac{H_a}{U_c}\right)\left.\frac{m\sinh mx}{\cosh mL}\right|_{x=L} = \left(T_b - T_a - \frac{H_a}{U_c}\right)\frac{m\sinh mL}{\cosh mL}$$

$$q = -tk\left.\frac{dT}{dx}\right|_{x=L} = -tkm\left(T_b - T_a - \frac{H_a}{U_c}\right)\tanh mL \qquad tkm = \frac{U_c}{m^2}m = \frac{U_c}{m}$$

$$q = -\frac{U_c}{m}\left(T_b - T_a - \frac{H_a}{U_c}\right)\tanh mL = \frac{1}{m}\left[H_a - U_c(T_b - T_a)\right]\tanh mL$$

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Absorber Plate Analysis VII

- Account for heat flows into tube from two sides and define $F = \tanh(mL)/(mL)$

$$q_{plate} = \frac{2}{m}\left[H_a - U_c(T_b - T_a)\right]\tanh mL = 2LF\left[H_a - U_c(T_b - T_a)\right]$$

- Solar energy/heat loss above tube, $D[H_a - U_c(T_b - T_a)]$ is added to plate heat transfer
  - The total is the useful heat transfer to the fluid in the solar collector tubes

$$q_{total} = (2LF + D)\left[H_a - U_c(T_b - T_a)\right] = q_u'$$

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Absorber Tube Analysis

- Relationship between q'_u (useful heat per unit length), fluid temperature, T_f, and bond temperature T_b

$$q_u' = \frac{T_b - T_f}{\frac{1}{C_B} + \frac{1}{h_{c,i}\pi D_i}}$$

  - C_B = bond conductance > 35 W/m·K
    - $C_B = k_B w_B / t_B$ where k_B, w_B, and t_B are bond thermal conductivity, width and thickness
  - $h_{c,i}$ = Heat transfer coefficient inside tube
  - D_i = inside tube diameter

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Absorber Tube Analysis II

- Can eliminate bond temperature between two equations for q'_u

$$q'_u = \frac{T_b - T_f}{\frac{1}{C_B} + \frac{1}{h_{c,i}\pi D_i}} \Rightarrow T_b = T_f + q'_u\left(\frac{1}{C_B} + \frac{1}{h_{c,i}\pi D_i}\right)$$

$$q'_u = (2LF + D)\left[H_a - U_c(T_b - T_a)\right]$$

$$q'_u = (2LF + D)\left[H_a - U_c\left(T_f + q'_u\left(\frac{1}{C_B} + \frac{1}{h_{c,i}\pi D_i}\right) - T_a\right)\right]$$

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Absorber Tube Analysis III

$$q'_u\left[1 + (2LF + D)U_c\left(\frac{1}{C_B} + \frac{1}{h_{c,i}\pi D_i}\right)\right] = (2LF + D)\left[H_a - U_c(T_f - T_a)\right]$$

- Divide by $U_c(2LF + D)$ and solve for q'_u

$$q'_u\left[\frac{1}{(2LF + D)U_c} + \left(\frac{1}{C_B} + \frac{1}{h_{c,i}\pi D_i}\right)\right] = \frac{1}{U_c}\left[H_a - U_c(T_f - T_a)\right]$$

$$q'_u = \frac{\frac{1}{U_c}\left[H_a - U_c(T_f - T_a)\right]}{\left[\frac{1}{(2LF + D)U_c} + \left(\frac{1}{C_B} + \frac{1}{h_{c,i}\pi D_i}\right)\right]} = wF'\left[H_a - U_c(T_f - T_a)\right]$$

$w = 2L + D$

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Absorber Tube Analysis IV

- F' = Collector efficiency factor
- w = Distance between tube centerlines

$$F' = \frac{\text{Thermal Resistance Between Plate and Ambient}}{\text{Thermal Resistance Between Fluid and Ambient}}$$

$$F' = \frac{\frac{1}{U_c}}{w\left[\frac{1}{(2LF + D)U_c} + \left(\frac{1}{C_B} + \frac{1}{h_{c,i}\pi D_i}\right)\right]}$$

- Now find fluid temperature increase from inlet to exit

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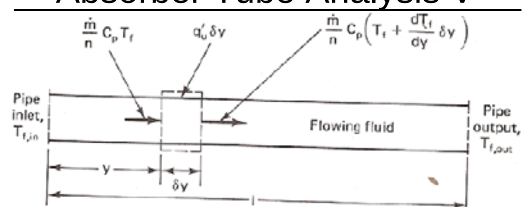
Absorber Tube Analysis V



- Energy balance over differential δy in one of n tubes where added heat is $q'_u\delta y$ and $\dot{m}$ is total mass flow rate

$$\frac{\dot{m}}{n}c_p\frac{dT_f}{dy} = q'_u = wF'\left[H_a - U_c(T_f - T_a)\right]$$

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Absorber Tube Analysis VI

- Rearrange to separate variables and integrate $\frac{\dot{m}}{n}c_p\frac{dT_f}{dy} = wF'\left[H_a - U_c(T_f - T_a)\right]$

$$\int_{T_{f,in}}^{T_{f,out}} \frac{dT_f}{T_f - T_a - H_a/U_c} = -\frac{nwF'U_c}{\dot{m}c_p}\int_0^{\ell} dy$$

$$\ln\left(\frac{T_{f,out} - T_a - H_a/U_c}{T_{f,in} - T_a - H_a/U_c}\right) = -\frac{nwF'U_c}{\dot{m}c_p}\ell = -\frac{A_cF'U_c}{\dot{m}c_p}$$

- nwl = (number of tubes)(distance between tubes)(length of tube) = collector area = A_c

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Absorber Tube Analysis VII

$$\frac{T_{f,out} - T_a - H_a/U_c}{T_{f,in} - T_a - H_a/U_c} = e^{-\frac{A_cF'U_c}{\dot{m}c_p}}$$

- Introduce F_R = (Actual heat transfer) / (Heat transfer is entire plate is at $T_{f,in}$

$$F_R = \frac{\dot{m}c_p(T_{f,out} - T_{f,in})}{A_c\left[H_a - U_c(T_{f,in} - T_a)\right]} = \frac{\dot{m}c_p}{U_cA_c}\frac{(T_{f,out} - T_{f,in})}{(H_a/U_c - T_{f,in} + T_a)}$$

$$F_R = \frac{\dot{m}c_p}{U_cA_c}\frac{(T_{f,out} - T_a - H_a/U_c) - (T_{f,in} - T_a - H_a/U_c)}{-(T_{f,in} - T_a - H_a/U_c)}$$

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Absorber Tube Analysis VIII

$$F_R = \frac{\dot{m}c_p}{U_c A_c}\left[1 - \frac{\left(T_{f.out} - T_a - H_a/U_c\right)}{\left(T_{f.in} - T_a - H_a/U_c\right)}\right] = \frac{\dot{m}c_p}{U_c A_c}\left(1 - e^{-\frac{A_c F' U_c}{\dot{m}c_p}}\right)$$

- Last step uses previous result $\frac{T_{f,out} - T_a - H_a/U_c}{T_{f,in} - T_a - H_a/U_c} = e^{-\frac{A_c F' U_c}{\dot{m}c_p}}$
- Definition: $F_R = \frac{\dot{m}c_p\left(T_{f.out} - T_{f.in}\right)}{A_c\left[H_a - U_c\left(T_{f.in} - T_a\right)\right]} = \frac{\dot{Q}_u}{A_c\left[H_a - U_c\left(T_{f.in} - T_a\right)\right]}$
- Result called Hottel-Whillier-Bliss Equation

$$\dot{Q}_u = F_R A_c\left[H_a - U_c\left(T_{f.in} - T_a\right)\right]$$

Summary of Results

- Q_u = useful heat transfer to working fluid

$$F' = \frac{1}{\frac{w}{U_c}\left[\frac{1}{U_c(2LF + D)} + \frac{1}{C_B} + \frac{1}{\pi D_i h_i}\right]}$$

$$F_R = \frac{\dot{m}c_p}{U_c A}\left[1 - e^{-\frac{U_c A F'}{\dot{m}c_p}}\right]$$

$$Q_u = AF_R\left[H_a - U_c\left(T_{f,in} - T_a\right)\right] \qquad T_{f,out} = T_{f,in} + \frac{Q_u}{\dot{m}c_p}$$

Solar Input

- H_a used previously is solar energy absorbed by the collector
- H_i is the solar radiation incident on the collector
- Two sources of solar radiation
  - Direct radiation from the sun
  - Diffuse radiation from atmosphere and ground reflection
- Direction affects amounts transmitted and absorbed

Transmission and Absorption

- Radiation entering top glass cover can be transmitted, absorbed and reflected
- Amount transmitted to second glass cover can also be transmitted, absorbed, and reflected
- Want overall proportion of incident radiation that is absorbed by collector
- This is given by $H_a = H_i\tau\alpha$, where $\tau\alpha$ is the mean of the transmissivity times the absorptivity for the total process

Collector Efficiency

- Instantaneous collector efficiency, η_c

$$\eta_c = \frac{Q_u}{A_c H_i}$$

- Average collector efficiency over a time period, τ, $\overline{\eta}_c$

$$\overline{\eta}_c = \frac{\int_0^\tau Q_u dt}{A_c \int_0^\tau H_i dt}$$

Solar Efficiency Testing

- Start with Hottel-Whillier-Bliss Equation

$$Q_u = F_R A_c\left[H_a - U_c\left(T_{f.in} - T_a\right)\right]$$

- Replace H_a by $H_i\tau\alpha$

$$Q_u = F_R A_c\left[H_i\tau\alpha - U_c\left(T_{f.in} - T_a\right)\right]$$

- Substitute into efficiency equation

$$\eta_c = \frac{Q_u}{A_c H_i} = \frac{F_R A_c\left[H_i\tau\alpha - U_c\left(T_{f.in} - T_a\right)\right]}{A_c H_i} = F_R\tau\alpha - \frac{F_R U_c\left(T_{f.in} - T_a\right)}{H_i}$$

Solar Efficiency Testing II

- Last equation shows how to determine collector parameters in testing

$$\eta_c = \frac{Q_u}{A_c H_i} = F_R \tau\alpha - \frac{F_R U_c \left(T_{f,in} - T_a\right)}{H_i}$$

  – Measure and plot collector efficiency, η_c, as a function of $(T_{f,in} - T_a)/H_i$
    - Measure $Q_u = \dot{m} c_p \left(T_{f,out} - T_{f,in}\right)$
    - Intercept is $F_R\tau\alpha$
    - Slope is $-F_R U_c$

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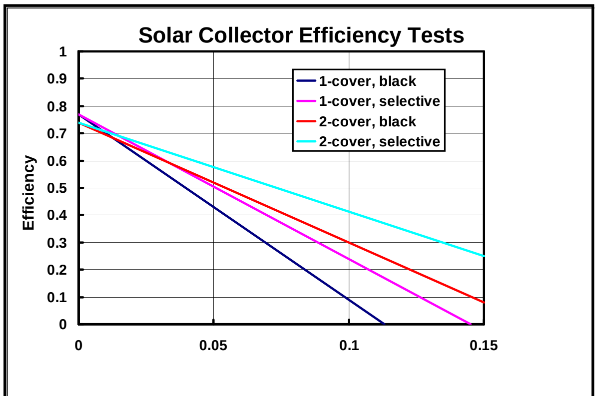

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Solar Efficiency Test Results

Type of Collector	Intercept = $F_R\tau\alpha$	Value at x = 0.1	Slope = $-F_R U_c$
1-cover, black	0.77	0.095	–6.75
1-cover, selective	0.77	0.23	–5.4
2-cover, black	0.74	0.30	–4.4
2-cover, selective	0.74	0.41	–3.3

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One Final Word

- Efficiency tests are usually done with the collector normal to the sun's rays
- This measures a particular $\tau\alpha$ product, called the normal $\tau\alpha$ product, $(\tau\alpha)_n$
- For actual collector operation the $\tau\alpha$ product can vary over the year with the position of the sun
- Adjustments can be made to account for this variation to the $\tau\alpha$ product

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